

# PROCEEDINGS OF SPIE

[SPIDigitalLibrary.org/conference-proceedings-of-spie](https://spiedigitallibrary.org/conference-proceedings-of-spie)

## Laser Interferometer Gravitational-Wave Observatory (LIGO) project: overview and status

Richard L. Savage

Richard L. Savage, "Laser Interferometer Gravitational-Wave Observatory (LIGO) project: overview and status," Proc. SPIE 3270, Methods for Ultrasensitive Detection, (15 May 1998); doi: 10.1117/12.308362

**SPIE.**

Event: Optoelectronics and High-Power Lasers and Applications, 1998, San Jose, CA, United States

# The Laser Interferometer Gravitational-Wave Observatory (LIGO) Project: overview and status

R. L. Savage, Jr.

LIGO Hanford Observatory, P.O. Box 1970, Mail Stop S9-02, Richland, WA 99352, USA

## ABSTRACT

The LIGO Project\* is a joint effort between the California Institute of Technology and the Massachusetts Institute of Technology to build and operate a novel astronomical observatory that directly senses gravitational waves, and in doing so open a new observational window to the universe. Construction is well underway at the two observatory sites: Hanford, Washington and Livingston Parish, Louisiana. Installation of detector components is planned to begin in the spring of 1998 with the first data run at the designed strain sensitivity of  $h \sim 2 \times 10^{-23} \text{m}/\sqrt{\text{Hz}}$  scheduled to begin in 2002.

**Keywords:** LIGO, gravitational wave, astronomy, interferometer

## 1. INTRODUCTION

The Laser Interferometer Gravitational-Wave Observatory (LIGO) Project<sup>1-3</sup> is a joint effort by the California Institute of Technology (Caltech) and the Massachusetts Institute of Technology (MIT). The principal goal of the project is the direct detection of gravitational waves emanating from astrophysical sources and, ultimately, utilization of the gravitational waves to open a new observational window on the Universe. LIGO will become part of an international network of gravitational-wave detectors. Anticipated members of the network are the joint UK and German GEO600 Project, the joint French and Italian VIRGO Project, the Japanese TAMA Project, the planned Australian ACIGA Project (the only detector planned for the southern hemisphere), and several groups operating resonant bar detectors in the US, Italy, and Australia.

The LIGO Project began as a collaboration between Caltech and MIT to design, build and carry out initial searches with a novel, large-scale, interferometric gravitational-wave detector, but is evolving into two distinct, but closely related entities: the LIGO Laboratory, and the LIGO Scientific Collaboration (LSC). The LIGO Laboratory, managed by Caltech and MIT, will install and operate the initial LIGO detector and run the observatories and support facilities. The LSC, an international collaboration which now includes over twenty five member groups, is responsible for development of advanced detectors, detector diagnostics, data analysis, and detector subsystems for future LIGO upgrades. In many cases, individuals and groups participate in both organizations.

## 2. GRAVITATIONAL WAVES

From the viewpoint of Sir Isaac Newton in 1687,<sup>4</sup> space was described by Euclidean geometry: absolute and rectilinear. He also perceived time as an absolute quantity that progresses in the same way for all observers. From Newton's perspective, gravity was a force that causes massive objects, positioned in absolute space, to attract. These views were widely accepted and were very successful in describing the physical world until the end of the nineteenth century when several observed phenomena were found to be incompatible with Newton's descriptions of space and time. It was Albert Einstein, who in the early twentieth century formulated new descriptions of space, time, and gravity that were able to both explain the newly observed phenomena and agree with Newtonian physics under the conditions where Newton's laws had proved so successful.

Einstein's theory of relativity describes space and time as being intimately related (actually one entity called spacetime) and not the same for all observers. According to Einstein, gravity is the result of curvature, deviations of spacetime from Euclidean geometry. His theory predicted that distortions in the fabric of spacetime could propagate at the speed of light in the form of gravitational waves. Although Einstein's relativity has in many ways been proven

---

Other author information: E-mail: [savage\\_r@ligo.caltech.edu](mailto:savage_r@ligo.caltech.edu)

\*The LIGO project is supported by the National Science Foundation under cooperative agreement PHY-9210038.



**Figure 1.** Artist's depiction of the ripples in spacetime generated by the in-spiral near coalescence of two black holes.

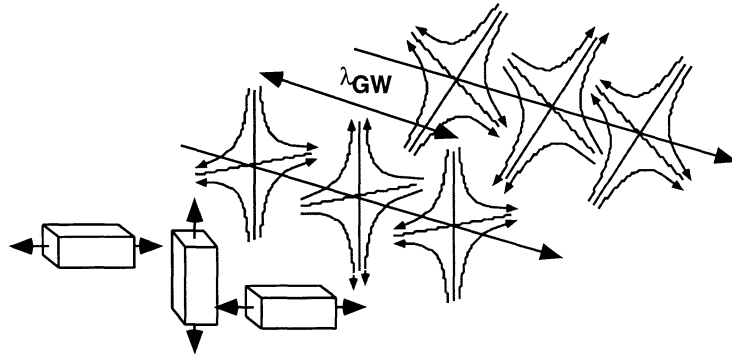
to be a more accurate description of the physical world, the gravitational waves he predicted more than eighty years ago have not yet been directly observed.

Gravitational waves have not yet been directly observed, but their existence has been indirectly confirmed. In 1974, Russell Hulse and Joseph Taylor<sup>5</sup> discovered the pulsar PSR 1913+16 which, together with its companion neutron star, constitute a “binary pulsar.” Hulse and Taylor recognized that this binary pulsar would provide a near-ideal test bed for many of the predictions of general relativity. Of the predictions they were able to confirm, one of the most significant was their observation that the orbital period of PSR 1913+16 varied precisely (to within the experimental accuracy of one percent) as predicted by general relativity for such a binary system losing energy via the emission of gravitational waves.<sup>6</sup> For this work, Hulse and Taylor were awarded the Nobel Prize for Physics in 1993.

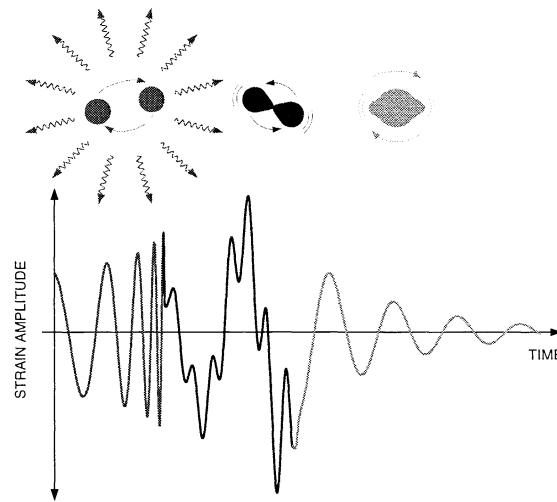
Gravitational waves are generated by non-spherically-symmetric accelerations of mass. In a multi-pole expansion of the mass distribution of a system, the temporal variations of the first few terms are zero because mass, momentum, and angular momentum are conserved. The first non-zero term (and in many cases, the dominant term) is the time variation of the gravitational quadrupole moment.<sup>7</sup> Thus a binary system, two stellar objects orbiting each other, emits gravitational waves. Figure 1 is an artist's depiction of spacetime curvature fluctuations generated by two in-spiraling black holes as they near coalescence.

Gravitational waves are transverse and induce a quadrupolar strain in space. The two polarizations, “plus” and “cross” are shown schematically in Figure 2. The apparent effect of a gravitational wave can be visualized by considering what happens to a sheet of rubber as it is stretched along one axis — the length along the perpendicular axis contracts. One half cycle later, the originally expanded direction contracts and the contracted direction expands, and so on. While one has to be careful separating space and time in this way according to general relativity, it can be simply stated that differences in the space-time intervals between pairs of free masses oriented perpendicular to each other, as measured by a beam of light, varies periodically in the presence of a gravitational wave. A laser interferometer with two perpendicular arms is thus a “natural” instrument for detecting gravitational waves.

Potential sources of gravitational waves that might be detected by LIGO include<sup>8</sup> non-axisymmetric gravitational collapse of stellar cores (e.g. Supernova 1987A, for example), in-spiral and coalescence of compact binary systems (e.g. neutron star-neutron star, neutron star-black hole, or black hole-black hole), and a stochastic background from primordial waves generated during the Big Bang (analogous with the cosmic microwave background radiation). Many believe that detection of unexpected events, phenomena which have not yet been imagined or predicted, is likely to be what LIGO will be best known for in the future.



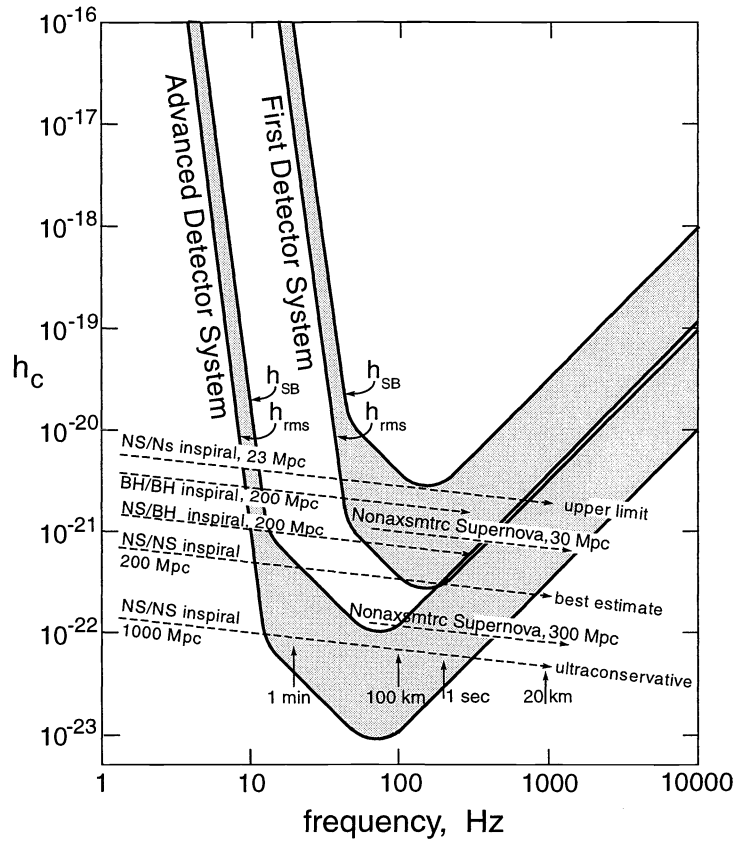
**Figure 2.** Forces felt by a “freely-falling” mass resulting from the passage of a gravitational wave (left). The two polarizations of the gravitational waves are  $h_+$ , the *plus* polarization (center), and  $h_\times$ , the *cross* polarization.



**Figure 3.** Prediction of waveform of gravitational radiation emitted by two coalescing black holes as they in-spiral, tidally disrupt each other and coalesce, then as the system rings down after coalescence.

Should LIGO and other efforts to directly sense gravitational radiation prove successful, we will be able to verify some of the last remaining details of Einstein’s general theory of relativity. Details such as the speed of propagation of gravitational waves and the polarizations of the waves, ergo the spin of the graviton. Perhaps more importantly, the direct observation of the gravitational waveform enables testing general relativity in the strong field regime.<sup>8</sup> Figure 3 shows the theoretically-predicted signal resulting from the in-spiral of two black holes. Initially, the signal grows in amplitude and increases in frequency (a *chirped* signal) as the black holes in-spiral. As they approach coalescence, they begin to tidally disrupt each other and enter the highly nonlinear, strong field regime where the waveform becomes extremely complicated. While general relativity provides the equations to predict the waveforms in this regime, even state-of-the-art supercomputers are challenged to generate the complex waveforms during the phase of tidal disruption and coalescence. Computer modeling of this phase is expected to yield results in the next few years, at about the time LIGO is planned to come on line. Comparison of the waveforms detected by LIGO with the computer models will enable a rigorous test of general relativity.

The challenge of the direct observation of gravitational waves lies primarily in the extremely weak interaction of gravitational radiation with matter. For example, the coalescence of a pair of 1.4 solar mass neutron stars at a distance of the nearest cluster of galaxies, the Virgo cluster, would result in a wave of strain amplitude,  $h_{\text{RMS}} \sim 10^{-21}$ . Even for a detector such as that planned initially for LIGO, incorporating interferometers with 4-km-long arms, the



**Figure 4.** Predicted signal strengths for binary in-spirals and LIGO initial and advanced detector sensitivities.

apparent root-mean-square (RMS) displacement induced by such a wave is only  $\sim 10^{-18}$  m. While the gravitational wave's weak interaction with matter poses difficult challenges for detection, it is also one reason that their detection is so valuable. It implies that they should propagate from source to detector virtually unimpeded by intervening matter. Conversely, many events that yield strong electromagnetic signals may be invisible to us because their signals are scattered by cosmic matter.

A plot of the theoretically predicted signal strengths from binary in-spirals and the LIGO initial and advanced detector sensitivities is shown in Figure 4<sup>†</sup>. The dashed lines are predicted signal levels based on estimates of the population density of coalescing binary systems. For neutron star-neutron star binaries, signal levels labeled *upper limit*, *best estimate*, and *ultraconservative*, are based on three estimates of the volume required to encompass three events per year. The frequency dependence of the signal strengths is a result of the dynamics of the signal waveform. The lower curved lines for both the first detector system and the advanced detector system are the expected detector noise levels; thus, a signal at these levels, if optimally oriented and polarized, would be detected with a signal-to-noise level of one. The upper traces for the first and advanced detectors are the result of averaging over all positions on the sky and all source polarizations and, assuming all non-Gaussian noise has been removed, are at a level where the “false” event rate from Gaussian noise is less than one per ten years. Figure 4 indicates that the first LIGO detector system will be just sensitive enough to confidently detect neutron star binary coalescences at the upper limit of source population density estimates. For this reason, a program is already underway to develop advanced detectors and detector subsystems that will increase LIGO's sensitivity. Eventually, even ultraconservative event rate predictions should be within reach of yielding three detectable events per year. Enhancement of the detector sensitivity is the principal aim of the LIGO Science Collaboration as described in Section 1.

<sup>†</sup>One parsec (pc) is a distance equivalent to 3.26 lightyears or approximately  $10^{17}$  m.

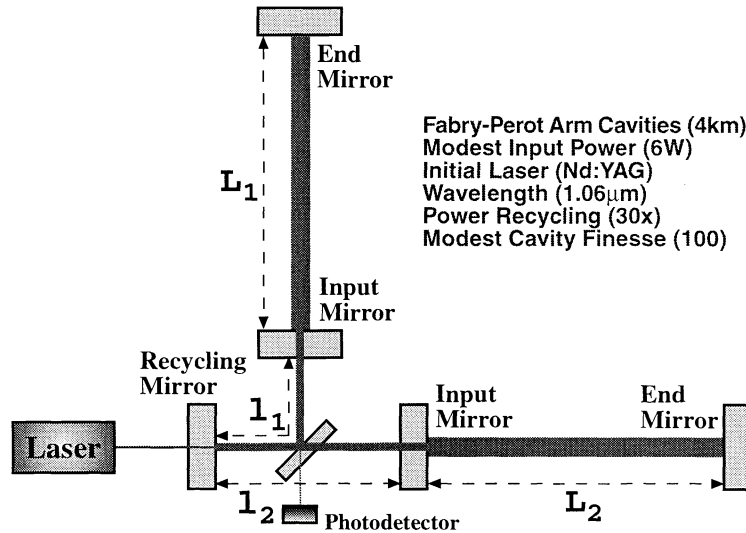


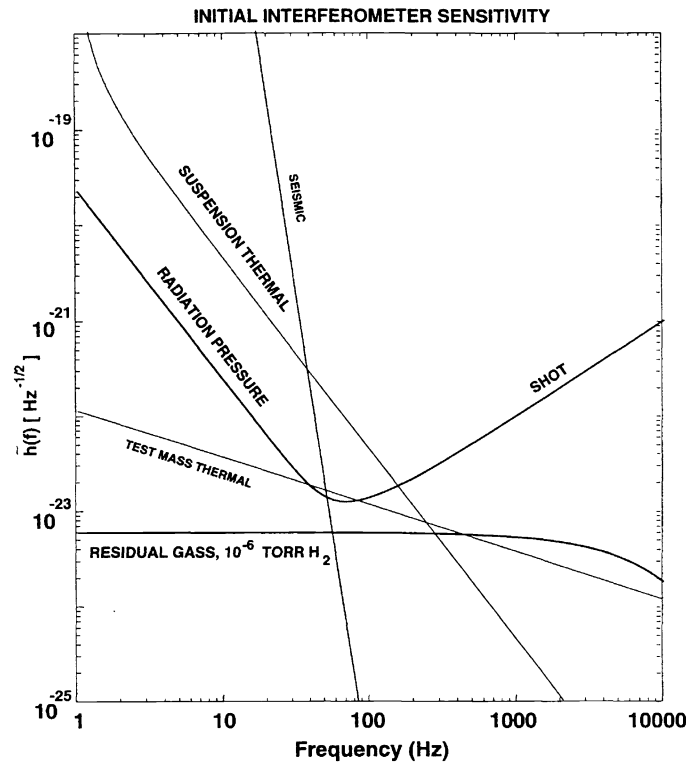
Figure 5. Schematic of one of the initial LIGO interferometers.

### 3. INTERFEROMETRIC GRAVITATIONAL-WAVE DETECTORS

A Michelson interferometer<sup>‡</sup> with suspended mirrors forms a *natural* instrument for detecting gravitational waves. The mirrors of the interferometer constitute test masses and are suspended to isolate them from ground vibrations and to make them “freely falling” in the two directions along the interferometer plane. The LIGO interferometers, shown schematically in Figure 5, are basically Michelson interferometers with two distinct variations: Fabry-Perot cavities in the arms and a recycling mirror that forms an additional resonant cavity. The Fabry-Perot arm cavities increase the sensitivity of the interferometers, effectively increasing the arm length. The laser light is stored in the arm, sampling the small length deviation many times before exiting. The interferometers are operated with the distance between optics adjusted so that the photo-detector is at a dark fringe, i.e. the light returning from the two interferometer arms destructively interferes in the direction of the photo-detector. The laser light that returns from the arms constructively interferes in the direction of the laser and would normally be lost from the interferometer. Because the detector sensitivity increases with increasing laser light power, a power recycling mirror is added. This mirror forms an additional resonant cavity by sending the rejected light back into the interferometer. The power incident on the beamsplitter is this increased by the recycling factor, which will be approximately 30 for the initial LIGO interferometers.

Noise sources affecting the performance of interferometric gravitational wave detectors can be grouped in two categories, displacement noise sources and sensing noise sources. Displacement noise sources actually cause relative motion of the test masses and include seismic noise transmitted through the seismic isolation systems and pendulum suspensions, thermally excited motions in the suspension pendulums, suspension wire violin modes, the internal vibrational modes of the test masses, and radiation pressure fluctuations caused by variations in the intensity of the laser light in the arms. Sensing noise sources do not actually cause relative motion of the test masses, rather they limit the precision with which one can sense the relative positions of the masses. They include frequency noise, intensity noise, and beam jitter on the incident laser radiation, index of refraction variations due to residual gas molecules traversing the inter-mass vacuum, shot noise due to counting statistics related to the quantum nature of light, and electronics noise in the many servo control systems for the interferometers. Estimates of the amplitude and frequency dependence of the principal noise source for the initial LIGO interferometers are plotted in Figure 6.

<sup>‡</sup>Other interferometer configurations, e.g. Sagnac interferometers or interferometers with arms set at relative angles other than 90 degrees, have also been proposed and studied. Most, if not all, of the gravitational-wave interferometers presently under construction utilize some variation of a Michelson interferometer.



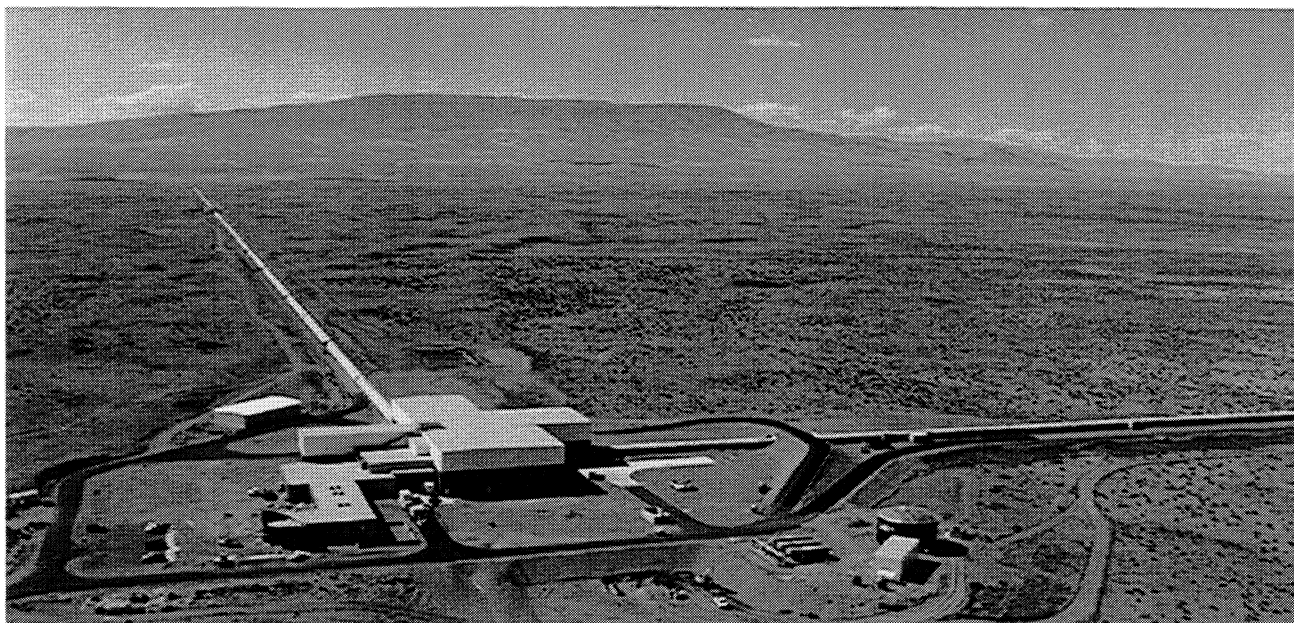
**Figure 6.** Estimates of amplitude and frequency dependence for the principal noise sources in the initial LIGO interferometers.

#### 4. LIGO DETECTOR IMPLEMENTATION

The initial LIGO detector will consist of three interferometers operating simultaneously and located at two observatory sites separated by approximately 3000 km. The site in Hanford, Washington will house two interferometers, one with 4-km-long arms and a half-length interferometer with 2-km-long arms. One 4-km-long interferometer will be installed at the site in Livingston, Louisiana. The LIGO sites have been chosen partly because they are widely separated, relatively flat and distant from sources of environmental noise. Figure 7 is an aerial photograph of the LIGO Hanford observatory site taken in December, 1997. The central group of buildings is the corner station which includes the laser and vacuum equipment area which houses the vacuum equipment and interferometer components and the attached, operations support building which houses the laboratories, offices and control room. The beam tube enclosures covering the north-west and south-west arms of the interferometers are also visible, and the mid-and end-stations of the south-west arm are visible in the distance. The installation of the vacuum chambers that will house the interferometer is scheduled to be completed during 1998. Construction at the Livingston site is phased after the Hanford site by approximately six months.

##### 4.1. Beam Tubes

One of the major efforts in the construction of the LIGO facilities has been the fabrication and installation of the beam tubes that provide an evacuated path between the suspended test masses at the ends of the interferometer arms. One-eighth inch thick, type 304 stainless steel sheet has been specially treated (baked at high temperature in air) to reduce hydrogen outgassing and spiral welded into 1.2 m diameter tube sections approximately 20 m in length. These beam tube sections have been cleaned, leak tested, and welded, along with inter-spaced bellows sections, into 2-km-long beam tube modules under clean room conditions. The beam tubes are covered by 5-inch-thick concrete arches in 10-foot-long sections to protect them from damage and the environment. Each beam tube module will be



**Figure 7.** Aerial photograph of the LIGO Hanford observatory showing the corner station and the southwest arm with the mid- and end-stations visible in the distance. The photograph was taken in September, 1997.

insulated and baked under vacuum to remove water and contaminants from the surface. They will be heated by passing a direct current through the tube and baked at 150 degrees Celsius for thirty days. After baking, hydrogen is expected (based on experiments with beam tube sections) to be the dominant residual gas in the tubes. With only end pumping by large, cylindrical liquid nitrogen traps, the hydrogen partial pressure is predicted to be below  $10^{-9}$  Torr. All of the beam tube modules at Hanford have been evacuated and leak tested and the north-west arm has been wrapped with insulation in preparation for the bake which is scheduled to begin by April, 1998.

#### 4.2. Vacuum Equipment

The vacuum chambers that will house the interferometer components will be located inside the corner, mid, and end station buildings. The station buildings have been sized to accommodate up to three interferometers at Hanford (two full length and one half length) and up to two interferometers at Livingston (both full length). Figure 8 is a photograph of several of the test mass chambers inside the corner station of the Hanford observatory. The workers are surveying the placement of the chambers. The installation of all of the vacuum equipment at Hanford is scheduled for completion by July, 1998.

#### 4.3. Detector Components

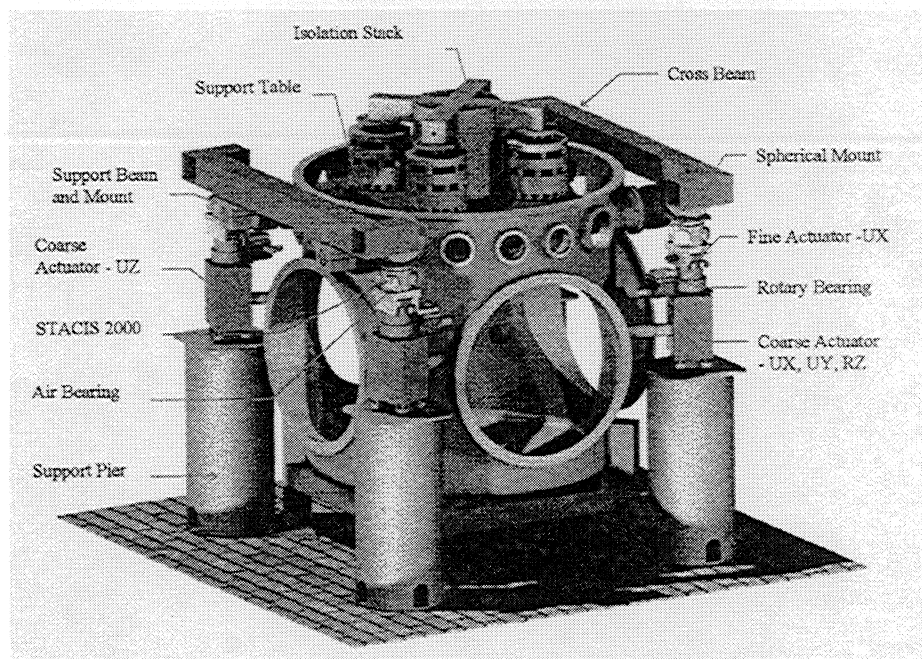
The main reason that the test mass chambers are so large is that they must accommodate seismically isolated optical tables. A computer-generated diagram of the seismic isolation system for the test mass chambers (being designed and fabricated by Hytec, Inc. in Los Alamos, NM) is shown in Figure 9. Inside and above the lower shell of the test mass chamber is the four-layer, passive vibration isolation stack that consists of masses (discs of stainless steel) and springs (constrained-layer-damped coil springs). The stack supports an optical platform from which the test mass is suspended. The stack itself is supported by beams that penetrate the chamber shell and rest on fine and coarse actuators that sit on tubular steel piers. The actuators enable compensation for Earth tides which distort the surface of the Earth by up to  $400\ \mu\text{m}$  over a 4 km baseline. An active isolation system attenuates the large (up to several  $\mu\text{m}$ ) motion at approximately 0.15 Hz due to the microseism. First-article tests of the vibrations isolation systems are scheduled to begin in February, 1998.

Because the interferometer arms are not perfectly matched, controlling fluctuations in the laser light is imperative. Relative power fluctuations in the laser light must be reduced to below  $10^{-7}\text{Hz}/\sqrt{\text{Hz}}$  over the gravitational wave

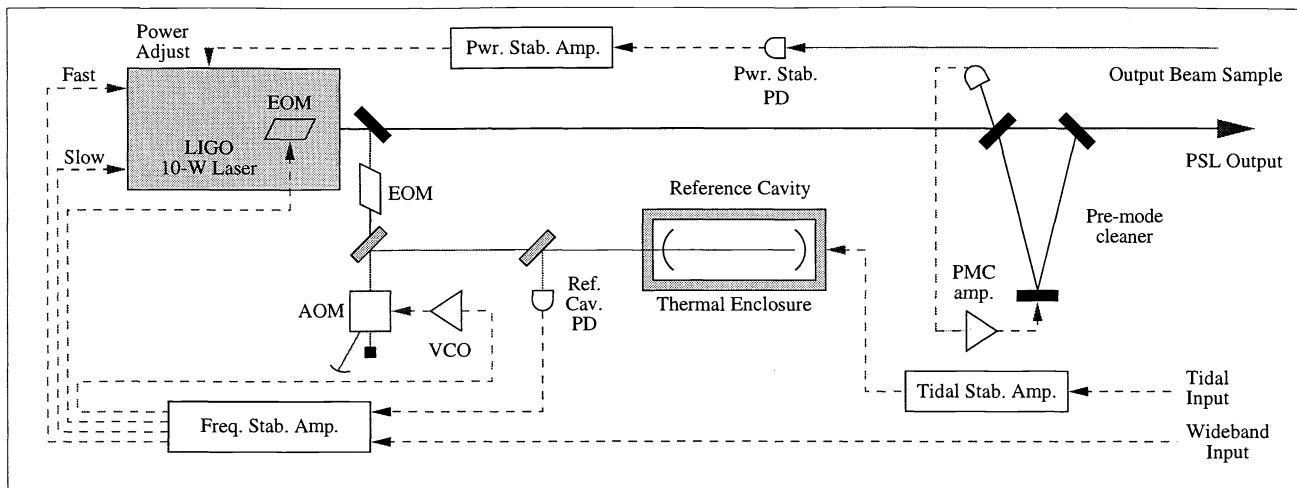




**Figure 8.** Photograph of surveyors measuring the locations of test mass chambers inside the corner station at the Hanford observatory. (December, 1997)



**Figure 9.** Drawing of seismic isolation system situated inside and around a test mass chamber.



**Figure 10.** Schematic diagram of the LIGO pre-stabilized laser system.

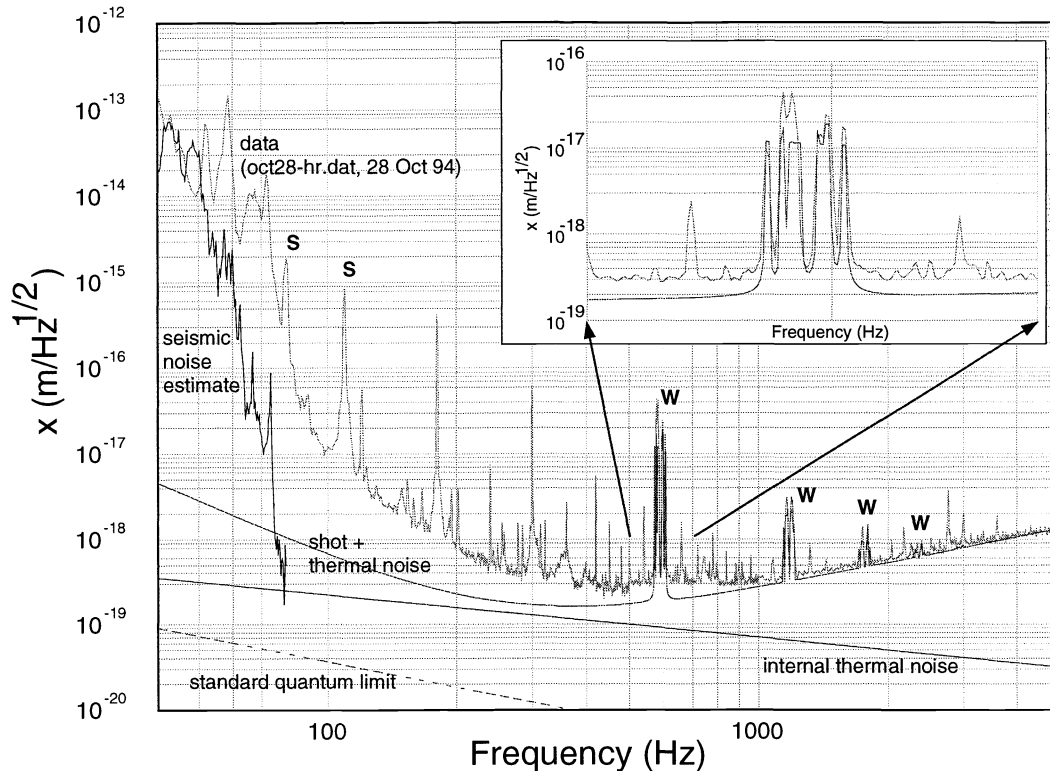
frequency band of interest, approximately 30 Hz to 10 kHz. Above the modulation frequency used for measuring the gravitational wave signal,  $\sim 25$  MHz, relative power fluctuations must be reduced to near the shot noise limit for 600 mW of detected power (less than  $10^{-9}/\sqrt{\text{Hz}}$ ). Frequency fluctuations must be reduced to less than  $10^{-7}\text{Hz}/\sqrt{\text{Hz}}$  in the gravitational wave band.

A schematic diagram of the LIGO pre-stabilized laser system<sup>9</sup> (PSL) is shown in Figure 10. The heart of the PSL is a 10-W laser source that is being developed under contract with LIGO by Lightwave Electronics, Inc. in Mountain View, CA. It is built in a master oscillator/power amplifier configuration. The first LIGO 10-W laser was delivered in January, 1998. Frequency fluctuations are reduced by a feedback control servo that will utilize three nested loops. The first loop reduces frequency fluctuations to less than  $10^{-1}\text{Hz}/\sqrt{\text{Hz}}$  by locking the laser to the resonance frequency of a Fabry-Perot reference cavity with a rigid, fused silica spacer suspended in vacuum. Frequency fluctuations are further suppressed by the second and third loops which utilize a 12-m-long, suspended-mirror, triangular mode cleaner and the 4-km-long interferometer arm cavities, respectively. Relative power fluctuations above 25 MHz are reduced by a triangular, fixed-spacer, ring cavity. A prototype PSL is being constructed at Caltech and is scheduled for delivery to Hanford in April, 1998.

The interferometer test masses are right circular cylinders fabricated from pieces of high-purity fused silica with bulk absorption of less than 5 ppm/cm. They are 25 cm in diameter, 10 cm thick, and weigh approximately 10 kg. The masses are suspended with single loops of 0.012 in. diameter steel music (piano) wire. The flat surfaces of the test masses are polished and coated to form pristine optical surfaces. Interferometer performance requirements dictate that the micro-roughness of the surfaces be less than a few angstroms in order to limit scattering losses and that the RMS surface figure (after subtracting power and astigmatism) be less than  $\lambda/1000$  (ten angstroms!) over the central 8 cm diameter. The test masses are being polished by CSIRO in Australia and General Optics in Moorpark, CA. Presently, most of the optical surfaces have been polished. Surface figures of  $\lambda/1000$  (RMS) over the full 20 cm beam aperture have been achieved.

The optical surfaces also require very low loss ( $\leq 1$  ppm), very high uniformity coatings over much larger areas than were commercially available. The coatings are being applied by the ion beam sputtering technique by Research Electro-Optics in Boulder, Colorado. In the first full-size coating demonstration, RMS surface figure of  $\sim \lambda/1000$  over the central 8 cm diameter ( $\lambda/200$  peak-to-valley over the central 20 cm diameter) with losses of less than 1 ppm were achieved. The first finished test masses are scheduled to arrive at the Hanford site by summer, 1998.

In addition to all of the optical, mechanical and vacuum hardware, the LIGO interferometers require a complex system of feedback control loops which utilize very low noise electronics to orient and control the positions of the optical components. In particular, five degrees of freedom must be controlled for many suspended optics that include, for each interferometer: four test masses, one beamsplitter, one recycling mirror, two fold mirrors (for the 2-km interferometer, three 12-m mode cleaner mirrors, and optics for mode matching telescopes.



**Figure 11.** Displacement noise spectrum of the 40-M Interferometer at Caltech

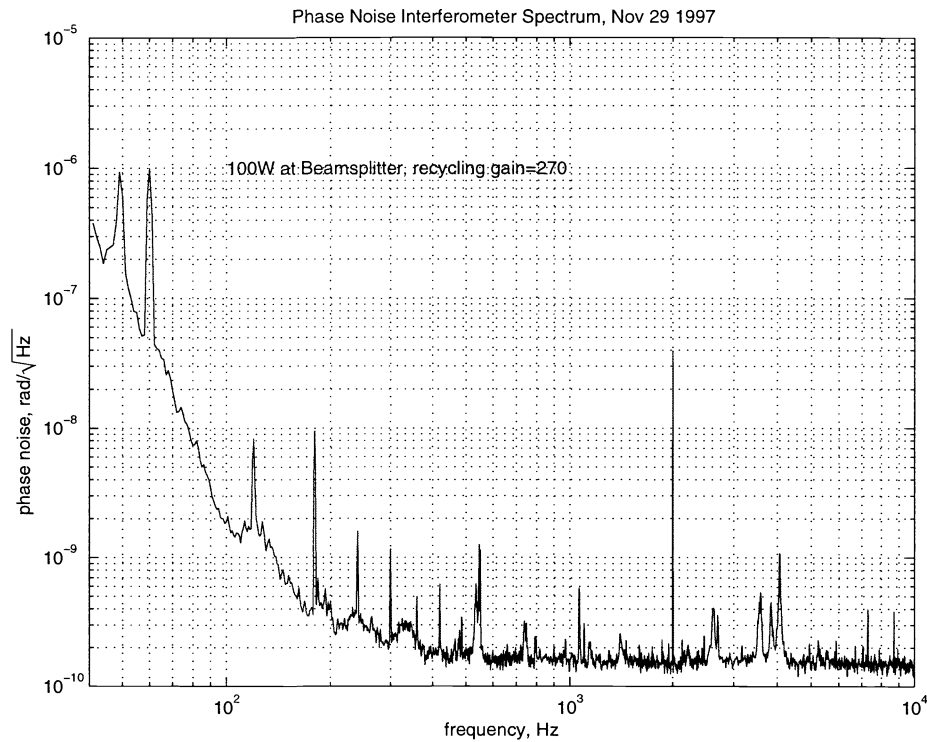
#### 4.4. Status of Prototype Interferometers

Two suspended-mirror prototype interferometers have been constructed by the LIGO project. The 40-m Interferometer at Caltech has been operating for many years and was designed principally to investigate sources and control of displacement noise. Figure 11 is a plot of the displacement sensitivity of the 40-m Interferometer in an earlier configuration. The data is compared with predictions based on modeling for the various noise sources. The peaks in the displacement spectrum labeled **S** are suspension resonances and the groups of peaks labeled **W** are excitations of the test masses driven by violin resonances in the suspension wires. The displacement spectrum noise drops to below  $3 \times 10^{-19} \text{m}/\sqrt{\text{Hz}}$  at 400 Hz which is close to the displacement sensitivity required for the initial LIGO interferometers. Above 1 kHz, the noise spectrum is dominated by sensing noise. This instrument has recently been configured in a similar fashion to the initial LIGO interferometers, a power recycled Michelson with Fabry-Perot arm cavities.

The other LIGO prototype interferometer, the Phase Noise Interferometer located at MIT, was been designed to investigate sensing noise. In order to achieve the design sensitivity, LIGO will require a phase sensitivity (sometimes referred to as the ability to “split” the fringe) of  $\leq 10^{-10} \text{radians}/\sqrt{\text{Hz}}$ . Figure 12 is a plot of the phase noise spectrum of the Phase Noise Interferometer recorded in late November, 1997. With 100 W of laser power incident at the beamsplitter (recycling gain of  $\sim 270$ ) the phase sensitivity is at the  $2 \times 10^{-10} \text{radians}/\sqrt{\text{Hz}}$  level down to several hundred hertz. The narrow spike at 2 kHz is a calibration spike and many of the other spikes are line-related.

### 5. PROJECT STATUS

The dominant activities for the LIGO Project the from present until the first data runs are listed in Table 1. Construction of the buildings and support facilities at the Hanford site is virtually complete, the beam tubes are all installed and have been tested for leaks. The baking of the beam tubes is scheduled to begin in April, 1998. All of the vacuum equipment has been installed along one arm and most of the vacuum hardware in the corner station has also been installed. Interferometer installation is scheduled to commence in April, 1998, beginning with the pre-stabilized



**Figure 12.** Phase noise spectrum of the Phase Noise Interferometer at MIT, November 29, 1997.

**Table 1.** Principal observatory activities from the present until the initial LIGO data run in 2002.

YEAR	ACTIVITY
1997	Facility Construction
1998	Begin Interferometer Installation
1999	Complete Interferometer Installation
2000	Commission Detectors
2001	Engineering Tests
2002	Begin Initial LIGO Data Run

laser system. LIGO science and engineering teams are busy preparing and testing interferometer subsystems and hardware is beginning to arrive at the observatory sites. Lots of hard work lies ahead, but it appears that operation at the design sensitivity of  $h \sim 2 \times 10^{-23} \text{m}/\sqrt{\text{Hz}}$  is within reach. The LIGO Science Collaboration, which will hold its second semi-annual meeting at Hanford in March, 1998, is busy investigating subsystem upgrades and advanced detectors for improved sensitivity.

## ACKNOWLEDGMENTS

The work reported here is that of the entire LIGO team, past and present. The author gratefully acknowledges enlightening discussions with F. J. Raab and the assistance of P. J. King with the preparation and editing of this manuscript. He also thanks D. J. Vieira for his invitation and encouragement to participate in this interesting conference session.

## REFERENCES

1. A. Abramovici et al., "LIGO: The Laser Interferometer Gravitational-Wave Observatory," *Science* **256**, pp. 325–333, 1992.
2. D. Coyne, "The Laser Interferometer Gravitational-Wave Observatory (LIGO) Project," in *Proceedings of the 1996 IEEE Aerospace Applications Conference, Snowmass at Aspen, Colorado*, vol. 4, pp. 31–61, 1996.
3. J. K. Blackburn, "The Laser Interferometer Gravitational-Wave Observatory Project **LIGO**," in *Mathematics of Gravitation, Gravitational Wave Detection*, A. Krolak, ed., *Polish Academy of Sciences Institute of Mathematics, Banach Center Publications* **41, Part II**, pp. 95–135, 1997.
4. I. Newton, *The Mathematical Principles of Natural Philosophy*, ed. F. Cajori, University of California Press, Berkeley, 1947.
5. R. A. Hulse and J. H. Taylor, "Discovery of a pulsar in a binary system," *Astrophys. J.* **195**, pp. L51–L53, 1975.
6. J. H. Taylor and J. M. Weinberg, "Further experimental tests of relativistic gravity using the binary pulsar PSR 1913+16," *Astrophys. J.* **345**, pp. 434–450, 1989.
7. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H. Freeman, New York, 1970.
8. K. S. Thorne, "Gravitational radiation," in *300 Years of Gravitation*, S. W. Hawking and W. Israel, eds., pp. 330–458, Cambridge University Press, 1987.
9. R. L. Savage, P. J. King, and S. U. Seel, "A Highly-Stabilized 10-Watt Nd:YAG Laser for the Laser Interferometer Gravitational-Wave Observatory (LIGO)," in *Proceedings of the Second International Symposium: Modern Problems of Laser Physics, Novosibirsk, Russia*, S. Bagayev and V. Denisov, eds., *Siberian Division of Russian Academy of Sciences*, 1997.